PROBABILISTIC SEISMIC RISK ASSESSMENT FOR COMPREHENSIVE RISK MANAGEMENT: MODELING FOR INNOVATIVE RISK TRANSFER AND LOSS FINANCING MECHANISMS

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ABSTRACT:

A specific catastrophic risk model has been developed to evaluate, building by building, the probabilistic losses and pure premiums of different portfolios, taking into account the seismic microzonation of cities. Understanding probable losses and reconstruction costs due to earthquakes creates powerful incentives for countries to develop planning options and tools to cope with risk, including allocating the sustained budgetary resources necessary to reduce those potential damages and safeguard development. This model has been used to evaluate the fiscal contingency liabilities of the government and to build an optimal structure for risk transfer and retention, considering contingent credits, reserve funds, insurance/reinsurance, and cat bonds. In addition, an innovative insurance mechanism has been implemented for private housing, using the estate-tax payment and covering the all low-income homeowners through cross subsidies. Lastly, the model allows the evaluation of an exceedance probability curve of cost-benefit ratio, providing an innovative and ground-breaking tool for decision makers to analyze the net benefits of the risk mitigation strategies, such as earthquake retrofitting and seismic code enforcement. This paper describes the model and the derived abovementioned tools, using the results of loss scenarios and the strategies implemented in some earthquake prone urban centers.

KEYWORDS: Seismic risk, building damage, insurance mechanisms, cost-benefit analysis, contingent liabilities.

1. PROBABILISTIC EARTHQUAKE RISK MODEL

The frequency of catastrophic seismic events is particularly low; this is one of the reasons why very limited historical data are available. Considering the possibility of future highly destructive events, risk estimation has to focus on probabilistic models which can use the limited available information to best predict future scenarios and consider the high uncertainty involved in the analysis. Therefore, risk assessments need to be prospective, anticipating scientifically credible events that might happen in the future. Seismological and engineering bases are used to develop earthquake prediction models which permit to assess the risk of loss as a result of a catastrophic event. Since large uncertainties are inherent in models with regard to event severity and frequency characteristics, in addition to consequent losses caused by such events, the earthquake risk model is based on probabilistic formulations that incorporate this uncertainty into the risk assessment. The probabilistic risk model...
(PRM) built upon a sequence of modules, quantifies potential losses arising from earthquake events as shown in the Fig. 1.

![Figure 1 Probabilistic earthquake risk model (PRM)](image)

2. **SEISMIC HAZARD MODULE**

The hazard module defines the frequency and severity of a peril, at a specific location. This is completed by analyzing the historical event frequencies and reviewing scientific studies performed on the severity and frequencies in the region of interest. Once the hazard parameters are established, stochastic event sets are generated which define the frequency and severity of thousands of stochastic events. This module can analyze the intensity at a location once an event in the stochastic set has occurred, by modeling the attenuation of the event from its location to the site under consideration, and evaluates the propensity of local site conditions to either amplify or reduce the impact.

The seismic hazard is expressed in terms of the exceedance rates of given values of seismic intensity \(a\). Its calculation includes the contribution of the effects of all seismic sources located in a certain influence area. Once these seismic sources are identified, a certain occurrence model is assigned to the earthquakes that take place there. In the most cases all seismic sources are modeled to follow a Poisson process in which \(\lambda(M)\) represents the activity rates for each faulting system. Since the seismic sources are volumes and the methodology considers a point source approach, the epicenters cannot only occur in the centers of the sources, but can also occur, with equal probability, in any point inside the corresponding volume. Therefore, for the simulation of event sets, sub-sources are defined by subdividing the seismic sources, depending on hypocentral distance \((R_0)\), in diverse geometric shapes. For each subdivision the seismicity of the source is considered to be concentrated in its center of gravity.

In addition the model considers the attenuation effects of the seismic waves by means of probabilistic spectral attenuation laws that include different source types and the local amplification effects based on microzonation studies and other available complementary information. Since the computed intensity is regarded as a random variable with lognormal distribution, its corresponding uncertainty value \(\sigma_Lna\) is considered to include the associated variability.

Assuming that the intensity variable has a lognormal distribution given the magnitude \((M)\) and distance \((R_0)\), the probability of a given seismic intensity \((a)\), \(Pr(A>a|M, R_0)\) is calculated as follows:

\[
Pr(A > a | M, R_0) = \Phi \left( \frac{1}{\sigma_{Ina}} \ln \frac{MED(A | M, R_0)}{a} \right)
\]

(2.1)

where \(\Phi(\cdot)\) is the standard normal distribution, \(MED(A | M, R_0)\) is the median value of the intensity variable (given by the corresponding attenuation law) and \(\sigma_{Ina}\) the standard deviation of the natural logarithm of the intensity \((a)\). This methodology based on (Esteva, 1970; Ordaz, 2000), generates stochastic seismic events at random locations within the modeled seismic sources, calculates the probability density function (PDF) of the seismic intensity \((a)\) for a specific location, and, if required, adds up the contributions of all sources and
magnitudes in order to compute intensity exceedance rates, as those depicted in Fig. 2. From these intensity exceedance rates, it is possible to determine uniform hazard spectra (UHS) for a specific site, based on the calculated intensity value (i.e. spectral acceleration) associated to a fixed return period. Therefore UHS can be determined by connecting the intensity points calculated from Fig. 2 for a given exceedance rate (inverse of the return period) for different structural periods \( T \).

![Figure 2: Bedrock site exceedance rates for different structural periods in two cities of Colombia. ERN-Colombia, (2005a)](image)

If the procedure described is followed for different locations within the city, and the selected intensity variable is calculated for the 475 year period, it is possible to build maps for different seismic intensities at ground level.

### 3. EXPOSURE MODULE

The exposure values of “assets at risk” are estimated either from available secondary data sources such as existing databases or they are derived from simplified procedures based on general macroeconomic and social information such as population density, construction statistics or more specific information. This “proxy” approach is used when the preferred specific site by site data are not available. Based on the information available, a new input database is constructed based on GIS, and specific required information is completed. Table 3.1 summarizes the minimum information for analysis required by the system. Additional more detailed parameters can be introduced to the database in order to improve the results’ general reliability.

<table>
<thead>
<tr>
<th>Hazard</th>
<th>Exposure</th>
<th>Vulnerability</th>
<th>Retention/Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Department</td>
<td>Value at risk</td>
<td>Number of stories</td>
<td>Retention percentage</td>
</tr>
<tr>
<td>Municipality</td>
<td>Exposure limit</td>
<td>Construction class</td>
<td>Deductible</td>
</tr>
<tr>
<td>Address</td>
<td>Building</td>
<td>Construction year</td>
<td>Coinsurance</td>
</tr>
<tr>
<td>GPS coordinates</td>
<td>Contents</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Special routines allow for the visualization of the database information and general interpretation indices are calculated. Fig. 3 presents example maps of Bogotá’s database used for analyzing all building constructions in the city, building a model of up to 1 million items.

<table>
<thead>
<tr>
<th>Building use</th>
<th>Total persons</th>
<th>Persons inside building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Day [%]</td>
<td>Night [%]</td>
</tr>
<tr>
<td>Residential</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>Commercial, Industrial, other</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>Health</td>
<td>15 m²/person</td>
<td>100</td>
</tr>
</tbody>
</table>

In order to calculate the social impact, general information related to building occupation is also estimated. Maximum occupancy and occupancy percentage at different hours are also defined in order to allow different time scenarios of the event’s occurrence. When no specific occupation information is available, approximate density occupation by construction class can be used in order to complete such information. Table 3.2 presents some of the reference information used for general occupancy estimation in medium-sized Colombian cities.
4. VULNERABILITY MODULE

The module quantifies the damage caused to each asset class by the intensity of a given event at a site (Miranda, 1999). The development of asset classification is based on a combination of construction material, construction type (say, wall & roof combination), building usage, number of stories and age. Estimation of damage is measured in terms of the mean damage ratio (MDR). The MDR is defined as the ratio of the expected repair cost to the replacement cost of the structure. A vulnerability curve is defined relating the MDR to the earthquake intensity which can be expressed in terms of maximum acceleration (e.g. useful for 1-2 story buildings), spectral acceleration, velocity, drift or displacement (e.g. useful for multi-story buildings) at each location. Given a value of seismic intensity for a certain building type, MDR can be calculated using Eq. 4.1 (Miranda, 1999; Ordaz, 2000).

\[
E(\beta | \gamma_i) = 1 - \exp \left[ \ln 0.5 \left( \frac{Y_d}{Y_0} \right)^{\gamma} \right] 
\]

(4.1)

Specific vulnerability curves can be defined for building contents and for business interruption (BI) costs. A total of 20 construction classes are included in the system as detailed in Table 4.1 and Fig. 4. The system also allows for the use of customized vulnerability models.

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Adobe / tapial earth walls</td>
<td>10</td>
<td>Semi-confined masonry</td>
</tr>
<tr>
<td>2</td>
<td>Bahareque</td>
<td>11</td>
<td>Confined masonry</td>
</tr>
<tr>
<td>3</td>
<td>Non-reinforced masonry - without diaphragm (1 story)</td>
<td>12</td>
<td>Reinforced masonry</td>
</tr>
<tr>
<td>4</td>
<td>Non-reinforced masonry - rigid diaphragm (1 story)</td>
<td>13</td>
<td>Waffle/Flat slab</td>
</tr>
<tr>
<td>5</td>
<td>Non-reinforced masonry (&gt;2 stories)</td>
<td>14</td>
<td>Weak reinforced concrete frames with masonry infill</td>
</tr>
<tr>
<td>6</td>
<td>1st and 2nd floor non-reinforced masonry</td>
<td>15</td>
<td>Strong reinforced concrete frames with masonry infill</td>
</tr>
<tr>
<td>7</td>
<td>Light roof warehouse and non-reinforce. masonry walls</td>
<td>16</td>
<td>Reinforced concrete frames with concrete shear walls</td>
</tr>
<tr>
<td>8</td>
<td>Light wood structure</td>
<td>17</td>
<td>Reinforced concrete shear walls</td>
</tr>
<tr>
<td>9</td>
<td>Church</td>
<td>18</td>
<td>Light roof warehouse, steel columns and masonry walls</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>Light roof warehouse, concrete columns and masonry walls</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>Steel frames</td>
</tr>
</tbody>
</table>
5. DAMAGE AND LOSS MODULE

To calculate losses, the damage ratio derived in the vulnerability module is translated into economic loss by multiplying the damage ratio by the value at risk. This is done for each asset class at each location. Losses are then aggregated as required (Ordaz et al., 1998; 2000). The loss module estimates the net losses taking into account the insurance information (e.g., deductible, sum insured). Risk measures produced by the model provide risk managers and decision makers with essential information required to manage future risks. One measure is the Average Annual Loss and the other is the Loss Exceedance Curve. Other measures, such as the Pure Risk Premium and the Probable Maximum Loss, can be computed based on the former.

- **Average Annual Loss.** AAL is the expected loss per year. Computationally, AAL is the sum of products of event expected losses and event annual occurrence probabilities for all stochastic events considered in the loss model. In probabilistic terms AAL is the mathematical expectation of the annual loss.

- **Pure Risk Premium.** PRP equals the AAL divided by the replacement value of the asset, usually expressed as a rate per mill of monetary value.

- **Loss Exceedance Curve.** LEC represents the annual frequency with which a loss of any specified monetary amount will be exceeded. This is the most important catastrophe risk metric for risk managers, since it estimates the amount of funds required to meet risk management objectives. The LEC can be calculated for the largest event in one year or for all (cumulative) events in one year. For risk management purposes, the latter estimate is preferred, since it includes the possibility of one or more severe events resulting from earthquakes. Fig. 5 presents some LEC results for cities en Colombia (CEDERI, 2005; ERN, 2005a/b).

- **Probable Maximum Loss.** PML represents the loss amount for a given annual exceedance frequency, or its inverse, the return period. Depending on a stakeholder’s risk tolerance, the risk manager may decide to manage for losses up to a certain return period (e.g. 1 in 300 years). For that stakeholder (e.g. a public or private agency), the PML is the 300-year loss. For others, it may be 150 years, or for others 500 years. It is
noteworthy that it is frequent to set program insolvency at the one in 150-year period to one in 200-year level, which roughly corresponds to the level of solvency required for BBB+ companies rated by S&P. However, other stakeholders (e.g. governments or regulation agencies) involved have chosen much longer return periods, such as the Mexican Insurance Commission, which uses a return period of 1500 years to fix solvency margins of insurance companies in Mexico.

6. RISK TRANSFER AND RETENTION MODULE

Catastrophic risks such as earthquake risk impose a dreadful threat not only for private insurers and reinsurers, but also for governments whom in turn are risk-takers for most of the uninsured and uninsurable risk. Therefore seismic risk models become powerful tools for government officials in economical and financial planning institutions. The retention and transfer of risk should be a planned and somewhat controlled process, given that the magnitude of the catastrophic problem will very likely exceed the governmental response and financial capacities, especially for third world countries. In that respect Colombia, after a history of positive efforts towards risk management at national and local level, has recently developed an earthquake vulnerability reduction program, with the financial assistance of the World Bank. In addition, a Disaster Deficit Index has been obtained for the country with the financial support of the Inter-American Development Bank (Marulanda et al. in these proceedings). Two of the components of the former program focus on seismic risk estimation and subsequent risk transfer and risk retention strategies for the cities of Bogotá and Manizales. (CEDERI 2005; ERN 2005a/b).

7. COST-BENEFIT ANALYSIS (CBA) MODULE

Cost-benefit analysis (CBA) applied to seismic risk analysis is a systematic procedure for evaluating decisions related to strategic risk management. A CBA is conducted depending on available information. A simplified five-step procedure adapted to seismic risk analysis from Smyth et al. (2004) is described below.

   Step 1: Specify the nature of the problem.
   Step 2: Determine the direct costs of mitigation measures.
   Step 3: Determine the benefits of mitigation alternatives.
   Step 4: Calculate attractiveness of mitigation alternatives.
   Step 5: Choose the best mitigation alternative.

Since the exact sequence of earthquakes, defined by their occurrence times and intensities, is unknown, the B/C ratio must be regarded as a random variable. Hence, its probability density function (PDF) must be calculated in order to evaluate the Net Present Value (NPV) of the lump sum of probable future losses and its variability, considering random occurrence of seismic events. Eq. 7.1 and Eq. 7.2 give the mean and the variance, respectively, of the NPV of the losses due to all future earthquakes, \( L \), in terms of the two first moments of the loss during a randomly chosen earthquake.

\[
E(L) = \frac{E(\beta_i)}{\gamma} \tag{7.1}
\]

\[
VAR(L) = \frac{VAR(\beta_i)}{2\gamma^2} \tag{7.2}
\]

where \( \beta_i \) is the annual loss, that is, the sum of all losses accumulated during one year, and \( E(\beta_i) \) is the pure premium; a value usually reported in risk analyses. It is interesting to note that, according to Eq. 7.2, the expected present value of the losses is the expected loss in one year divided by the discount rate. In principle, one could think that \( L \) is normally distributed, since it is computed as the sum of a potentially large number of random variables. However, for usual values of \( \gamma \) (2-5% per year), the number of terms that really contribute to \( L \) is small. We have verified, through Monte Carlo simulations, that normality is usually a poor approximation (Cardona et al 2006). Indeed every term in \( L \) is, approximately, the product of two Beta-distributed, independent random variables and the product of such two variables can also be approximated with a Beta distribution. We
would have in this case that $L$ is formed with the sum of random variables that are approximately Beta distributed and at limit we recognize a Gamma distribution for $L$; i.e. when the expected value of $L$ is much smaller than the maximum possible value of $L$. The simulation analyses for each specific problem were developed based on the following eight-step procedure:

- **Step 1**: Calculate a dense LEC for the portfolio of interest considering two scenarios: the current structural condition and a scenario considering retrofitted buildings. This is required in order to estimate benefits of retrofitting (loss reduction) by calculating loss values for random occurrence of seismic events.
- **Step 2**: Determine an exceedance rate value $\nu_0$. That is to choose the frequency of exceedance of loss = 0.
- **Step 3**: Calculate the cumulative distribution function (CDF) of the losses during the next random event.
- **Step 4**: Simulate random occurrence of seismic events over time following a Poisson process.
- **Step 5**: Calculate the probable losses in both scenarios (current condition and retrofitted) for the event time-history calculated in step 4.
- **Step 6**: Calculate NPV of the benefits (loss reduction due to retrofitting). Select a proper discount rate ($\gamma$) for the cash flow.
- **Step 7**: Repeat step 6 to generate enough statistics for the frequency analysis (5,000 simulations were calculated in our examples).
- **Step 8**: Carry out frequency analysis for the statistics calculated using adequate counting ranges and build the CDF of the benefits.

An example of application of this method is presented for the case study of Bogotá’s public building sectors, based on the project developed by CEDERI for the city of Bogotá and the World Bank in 2005. Table 7.1 presents the information related to each public building sector analyzed and the probabilistic B/C ratio results for different exceedance probabilities. Fig. 6 presents the results of the probabilistic CBA based on the simulations carried out for different portfolios of public assets property of the municipality of Bogotá.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Building Number</th>
<th>Building value (USD millions)</th>
<th>Retrofit costs (USD millions)</th>
<th>Probabilistic B/C ratio Exc. Prob.75%</th>
<th>Probabilistic B/C ratio Exc. Prob.50%</th>
<th>Probabilistic B/C ratio Exc. Prob. 25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security</td>
<td>36</td>
<td>60.7</td>
<td>10.8</td>
<td>1.18</td>
<td>1.77</td>
<td>2.40</td>
</tr>
<tr>
<td>Health</td>
<td>63</td>
<td>124.1</td>
<td>14.3</td>
<td>0.77</td>
<td>1.05</td>
<td>1.43</td>
</tr>
<tr>
<td>Education</td>
<td>14</td>
<td>3.0</td>
<td>0.2</td>
<td>0.54</td>
<td>0.95</td>
<td>1.58</td>
</tr>
<tr>
<td>Water Company</td>
<td>77</td>
<td>48.6</td>
<td>37.9</td>
<td>0.11</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>Electric Company</td>
<td>55</td>
<td>44.1</td>
<td>0.6</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Gas Company</td>
<td>2</td>
<td>6.5</td>
<td>1.2</td>
<td>0.15</td>
<td>0.29</td>
<td>0.52</td>
</tr>
<tr>
<td>Phone Company</td>
<td>64</td>
<td>773.9</td>
<td>4.6</td>
<td>0.83</td>
<td>1.13</td>
<td>1.50</td>
</tr>
<tr>
<td>Transportation</td>
<td>35</td>
<td>4.6</td>
<td>0.4</td>
<td>4.05</td>
<td>5.08</td>
<td>6.25</td>
</tr>
<tr>
<td>Administration</td>
<td>61</td>
<td>86.8</td>
<td>12.1</td>
<td>0.54</td>
<td>0.75</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Figure 6. Probabilistic Benefit/Cost ratio for public buildings in Bogotá
This methodological approach allows a probabilistic comparison of the expected benefits of a seismic risk mitigation plan based on building retrofitting; therefore this approach is very useful in planning an optimum benefit-cost risk mitigation strategy. In Fig. 6, as the probability for benefit-cost ratio greater than one (dotted line) increases, the retrofitting strategy yields beneficial for risk reduction purposes.

8. CONCLUSION

A catastrophic risk model has been developed to evaluate, building by building, risk metrics such as the Probabilistic Maximum Loss and the expected Average Annual Loss of different building portfolios, taking into account the seismic hazard and geotechnical site effects and the structural vulnerability of the different construction classes in urban centers. These metrics have been used to evaluate the fiscal sustainability and vulnerability of governments and it is useful to develop a financial optimal structure for risk retention and transfer taking into account contingent credits, reserve funds, insurance and reinsurance and cat bonds. An exceedance probability curve of benefit-cost ratio has been designed to analyze the net benefits of the risk mitigation strategies, such as earthquake retrofitting and seismic code enforcement, providing an innovative and ground-breaking tool for decision makers. These metrics and analyses have been applied in cities like Bogota and Manizales, Colombia, and they are being used by the cities governments to implement their risk retention and transfer structures to cover both public buildings and low-income housing of poor people throughout a cross-subsidizing strategy using estate-tax payments.

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